Mathematics & Statistics Know: Content and Concepts

PHASE THREE

Number | Mātauranga tau

By the end of this phase, students know that some **numbers have special properties**, such as primes, composites, squares, square roots, and cubes. A fraction can describe a **proportional relationship** between two amounts. Every fraction can be represented by an infinite set of equivalent fractions that occupy the same point on the number line. Multiplying a fraction by an **equivalent form** of 1, such as 3/3 , results in an equivalent fraction that can be useful for comparing, adding, and subtracting. **Decimals** continue the place-value system using negative powers of ten. They can be terminating, repeating and infinite, or non-repeating and infinite.

Students know that **integers** are positive and negative whole numbers, as well as zero. On a number line, fractions and decimals occur between integers. There are real-life situations described by quantities less than zero, such as temperature, below sea level, or debt, and these quantities can be operated on.

Students know that when calculating or forming expressions, the **order of operations** is important. Operations inside brackets (grouped together) are done first, then powers or exponents. If there are multiplication and division, these are done in left-to-right order; addition and subtraction are also done in left-to-right order. Students use the mnemonic GEMA: grouped, exponents, multiplication, addition. Division can result in a **remainder** expressed as a whole number, fraction, or decimal.

Algebra | Taurangi

By the end of this phase, students know that the **inverse property** applies to addition and multiplication. **Inequalities** can also include “or equal to” (≤, ≥) to show a relationship that allows for the possibility of equality. In algebra, a **variable** can be used to represent an unknown number, a quantity that can vary or change (e.g., y= 3x + 4, A = bh), or a specific unknown value to be solved for (e.g., 3a = 18). In algebra, there are conventional ways of writing multiplication and division.

Students also know that **linear patterns** have a constant rate of change and their XY graphs are **straight lines**. Not all patterns are linear. **Algorithms** help solve problems in a systematic way. Their instructions are created, tested, and revised.

Measurement | Ine

By the end of this phase, students know that in the **metric system** there are **base** measurements with **prefixes** added to show the size of units. A measurement can be converted from smaller to bigger units, and vice versa, by dividing or multiplying by powers of 10. Length is a **one-dimensional measure**, area is a **two-dimensional measure**, and volume is a **three-dimensional measure**. This is apparent in the notation of units, such as cm, cm2, and cm3. Shapes can be **decomposed** or **recomposed** to help us find measurements (e.g., for perimeters, areas, and volumes). When multiplying lengths or dividing an area or volume by a length, the result has a derived unit.

Geometry | Āhuahanga

By the end of this phase, students know that **spatial properties** of simple **polygons** and **polyhedra** can also apply to more complex two- and three-dimensional shapes.

Properties of two- and three-dimensional shapes that do not change under a transformation are called **invariant.**

Unknown angles can be found using the properties of angles on a **straight line,** angles at a **point, vertically opposite** angles, and interior angles in triangles and quadrilaterals.

Viewing objects from different angles gives **different perspectives**, which can be represented in models and diagrams. **Position, direction**, and **pathways** can be described using scale, compass points, and environmental features. **Coordinate systems** and maps can express position, direction, and pathways.

PHASE THREE

Statistics | Tauanga

By the end of this phase, students know that data collection and use involves a responsibility to protect the rights of people in relation to their own data and that of others, and the **ethical use** of data,including methods, interpretation, and conclusions. People need to understand who they are giving data to and why, before they agree to contribute to a dataset. The **statistical enquiry cycle** (PPDAC) can be used to conduct data-based investigations about the wider community. There are different types of **questions** used when undertaking statistical investigations: investigative (**summary, comparison, relationship, or time-series**), survey, data-collection, interrogative, or analysis questions. **Data visualisations** show patterns, trends, and variations. Alternative visualisations of the same data can lead to different insights and communicate different information. A **distribution** is formed from all the possible values of a variable and their frequencies. A **relationship** investigation looks for a relationship between paired numerical or paired categorical variables. **Predictions** or **assertions** may not be reflected in the data.

Probability | Tūponotanga

By the end of this phase, students know that a probability experiment involves repeated trials. Results from sets of repeated trials for the same probability experiment may vary. Some chance-based situations, such as the result of tossing a drawing pin, can only be explored by probability experiments. Estimates of probabilities from experiments should be based on a very large number of trials (the law of large numbers). The estimated probability of an event from an experiment equals the relative frequency for that event. If all possible outcomes in a chance-based situation are **equally likely**,
the probability of an event equals the number of ways the event can happen divided by the total number of possible outcomes. The statistical enquiry cycle (PPDAC) can be used to conduct experiments. For a given situation, **estimated probabilities** from **experiments** and **theoretical model** probabilities will differ. **Probability distributions** from experiments and probability distributions from theoretical models
will also differ.

Phase Three Number		
Number Structure		
Year 7	**Year 8**	**Teaching Methods/Representatives**
identify, read, write, compare, and order whole numbers using powers of 10(e.g., 10,000 = 10^4)	identify, read, write, compare, and order whole numbers and decimals using powers of 10 (e.g., 0.01 = 1 100 = 10^{-2})	place-value (PV) houses, number lines, inequality symbols
find the highest common factor (HCF) of two numbers under 100, and find the least common multiple (LCM) of two numbers under 10	use prime factorisation to represent a number and to find the HCF of two numbers	factor trees, systematic lists
use exponents to notate repeated multiplication, and identify square roots of square numbers up to at least 100	identify prime and composite numbers up to at least 100 and cube numbers up to at least 125	divisibility tests for composite and primenumbers demonstrations of exponent notation
use the mathematical processes to: – connect with divisibility rules, simplifying fractions, area, and volume – generalise conjectures about prime or composite numbers – investigate appropriate situations		
Operations		
use rounding and estimation to predict and to check the reasonableness of calculations	use rounding and estimation (including benchmarks) to predict and to check the reasonableness of calculations	› known facts › benchmarks
round whole numbers to any specified multiple of powers of 10, and round decimals to the nearest tenth, hundredth, or whole number		› number lines › visualising benchmarks
multiply whole numbers		› horizontal and vertical methods
divide whole numbers by 1- or 2-digit divisors (e.g., 327 ÷ 5 = 65.4 or 65 2 5)	divide whole numbers (e.g., 327 ÷ 15 = 21.8 or 21 4 5)	› horizontal and vertical methods
use the order of operations rule GEMA	use the order of operations rule GEMA	demonstrations of step-by-step layouts
order, compare, add, and subtract integers using tools	order, compare, add, and subtract integers	number lines, two sided counters, diagrams
use the mathematical processes to: – investigate situations where integers are used (e.g., temperature, altitude, profit and loss) – explain and justify findings using estimation, and checking using inverse operations		

Phase Three Number		
Rational Number		
Year 7	**Year 8**	**Teaching Methods/Representations**
identify, read, write, and represent fractions, decimals (to three places), and percentages	identify, read, write, and represent fractions, decimals, and percentages	bar models, number lines
compare, order, and convert between fractions, decimals (to three places), and percentages	compare, order, and convert between fractions, decimals, and percentages	› double number lines › benchmarks for demonstrating conversion › renaming to tenths or hundredths, or finding a common denominator
multiply and divide numbers by powers of 10	multiply and divide numbers by powers of 10	PV houses
find equivalent fractions, simplify fractions, and convert between improper fractions and mixed numbers	find equivalent fractions, simplify fractions, and convert between improper fractions and mixed numbers	› demonstrations of simplifying fractions and finding equivalent fractions using division and multiplication
multiply fractions and decimals by whole numbers, and find a percentage of a whole number	multiply fractions and decimals by whole numbers, and find a percentage of a whole number	horizontal and vertical methods › demonstrations of finding a percentage using multiplication
find a whole amount, given a simple fraction or percentage (e.g., ‘25% is $100, what is the original amount?’)	find a whole amount, given a simple fraction or percentage (e.g., ‘75% is $45, what is the original amount?’)	bar models
add and subtract fractions with different denominators up to tenths (e.g., 3/4 + 1/3)	add and subtract fractions with different denominators by using equivalent fractions	renaming using HCFs and LCMs
add and subtract decimals to three decimal places, with an emphasis on estimating before calculating	add, subtract, and multiply decimals, with an emphasis on estimating before calculating	horizontal and vertical methods
use proportional reasoning to explore relationships between quantities (e.g., ‘If there are 3 red for every 7 blue balls, how many balls are there altogether when there are 18 red balls?’)	use proportional reasoning to share in unequal proportions (e.g., ‘We have 100 stickers to share. for every 1 sticker I get, you get 3 stickers. How many do we each get?’)	diagrams and comparison models
use the mathematical processes to: – connect benchmarks (fractions, decimals, and percentages) and decimal operations with whole-number place values and operations – connect decimals with measuring – investigate HCFs and LCMs, the effect of multiplying and dividing decimals, situations where decimals are used and compared (e.g., sporting events), and proportional reasoning – explain and justify equivalence and which fraction is larger		

Phase Three Number		
Financial Maths		
Year 7	**Year 8**	**Teaching Methods/Represetations**
calculate costs, and change for any amount of money	create and compare weekly, monthly, and yearly finance plans (e.g., saving plans, phone plans, budgets, and 'buy now, pay later' services)	› spreadsheets › practical examples
calculate the percentage discounts of whole dollar amounts (e.g., 'What is 35% of $180?')	calculate percentage discounts	› demonstrations of finding a percentage and subtracting it from the whole, or of multiplying a whole by a required decimal fraction
use the mathematical processes to: – connect negative numbers with debt – investigate practical financial decisions and statistics in the media about growth or loss.		

Phase Three Algebra		
Generalising Number Properties		
Year 7	**Year 8**	**Teaching Methods/Representations**
explore multiplicative inverses (a number and its reciprocal) in multiplication		› fraction tiles, number lines › examples (for identifying trends)
explore additive inverses (pairs of opposites) in the addition and subtraction of positive and negative numbers (e.g., -6 + 8 = -6 + 6 + 2)	use commutative, associative, identity, and inverse properties with expressions, including those with negative numbers	› number lines
recall multiplication facts to at least 10 × 10 and identify and describe the divisibility rules for 2, 3, 5, 9, and 10	identify and describe the properties of prime and composite numbers and explore divisibility rules	› pattern exploration in 100s boards › multiplication grids
describe and use the commutative, distributive, and associative properties of operations (e.g., O × _ = _ × O)	simplify algebraic expressions involving sums, products, and differences, including by expanding single brackets expressions using the distributive property (e.g., 2(x + 3) + 1 = 2x + 6 + 1 = 2x + 7)	› physical manipulatives (e.g., algebra tiles) and digital manipulatives › systematic expansion approaches, including expansion tables
use the mathematical processes to: – represent algebraic expressions and equations using correct vocabulary and notation (e.g., 3 × b = 3b) – connect prime and composite numbers with factors, multiples, and divisibility rules – generalise relationships between positive and negative integers using the commutative, associate, and distributive properties of numbers – investigate appropriate situations		
Equations and Relationships		
form and solve 1-step linear equations (e.g., t + 7 = 12 ; 2s = 14)	form and solve 1- or 2-step linear equations (e.g., 5s – 3 = 17)	› word problems › demonstrations of equation-solving algorithms
find the value of an expression or formula given the values of variables (e.g., calculate w + 12 when w = 4)	find the value of an expression or formula given the values of variables	› variable values in practical situations with familiar formulae (e.g., for area, volume)
identify the constant rate of change and fixed value for a linear pattern, writing the equation using variables and algebraic notation to represent the rule, and using the rule to make predictions	determine if a pattern is linear and, if it is, write the equation for the pattern and use the equation	› tables and XY graphs › demonstrations of finding, and recording as an equation, the term-to-term and position- to-term rules for a sequence
use the mathematical processes to: – connect to measurement formulae – generalise a rule for a pattern and use this to justify a prediction of a term – investigate the history and use of growing patterns in tukutuku and other well-known patterns (e.g., the Fibonacci sequence)		

Phase Three Algebra		
Algorithmic Thinking		
Year 7	**Year 8**	**Teaching Methods/Representatives**
create, test, and revise algorithms involving a sequence of steps and decisions	create, test, revise, and use algorithms to identify, interpret, and explain patterns	› flowcharts, numbered step-by-step instructions, digital tools
	use the formula function of a spreadsheet to explore the effect of changing the value of a variable (e.g., hourly wages) on the results (i.e., cell values)	› spreadsheet cell conventions and formulae
use the mathematical processes to connect algorithms with methods for solving an operation.		

Phase Three Measurement		
Measuring		
Year 7	**Year 8**	**Teaching Methods/Representations**
estimate and then measure length, area, volume, capacity, mass (weight), temperature, data storage, time, and angle, using appropriate metric units		› rulers, scales, timers, protractors, thermometers, measuring jugs
select and use an appropriate base measure (e.g., metre, gram, litre) within the metric system, along with a prefix (e.g., kilo, centi) to show the size of units		› practical measuring › units that are appropriate for the situation
convert between metric units of length, mass (weight), and capacity, using whole numbers and decimals to express parts of a unit (e.g., 724g = 0.724kg)	convert between metric measurement units, including square units	› multiplying and dividing by powers of 10 › PV and decimal PV houses
find speed given distance and time	find distance given speed and time, or time given distance and speed	› bar models showing relationships between speed, distance, and time
use the mathematical processes to: – generalise equivalent measurements (e.g., 2.05L = 2050mL) – connect measurement conversions with multiplying and dividing by powers of 10 – investigate practical measurement situations, including reading tools with scales – explain which measurement tools and units are appropriate in a given situation		
Perimeter, Area, and Volume		
calculate the perimeter and area of compound shapes composed of triangles and rectangles	calculate the volume of triangular prisms and shapes composed of rectangular prisms	› demonstrations of finding perimeter, area and volume in practical situations › clear layouts for working
use the mathematical processes to: – generalise the formulae for finding the area of triangles and volume of triangular prisms – investigate practical contexts for finding perimeter, area, and volume		
Time		
read, interpret, and use timetables and charts that present measurement information		› demonstrations using a range of examples, including online apps calendars, timetables, and schedules.
convert between units of time and solve duration problems that involve fractions of time		› subtracting time to calculate for duration › inclusive counting
use the mathematical processes to: – generalise units of time using base-60 – investigate the duration of time in situations such as developing event schedules or planning journeys.		

Phase Three Geometry

Shapes

Year 7	Year 8	Teaching Method/Representation
classify shapes based on their properties, and name the resulting classes of shapes (e.g., triangles, pyramids)	describe triangles, quadrilaterals, and other polygons in relation to their side, diagonal, and angle properties	› a range of 2D and 3D shapes, including shapes that draw on tactile materials, diagrams, and digital tools
identify and describe angles at a point, angles on a straight line, and vertically opposite angles	reason about unknown angles in situations involving angles at a point, angles on a straight line, vertically opposite angles, interior angles of triangles, and polygons	› digital tools for exploring angles › measuring tools (e.g., rulers, protractors) › equations (to find unknown angles).
use the mathematical processes to: – generalise using angle rules to find unknown angles – investigate diagonals and angles of polygons – explain and justify classifications using flowcharts, Venn diagrams, and tables		

Spatial Reasoning

visualise, construct, and draw plan views for front, back, left, right, and top views of 3D shapes, using cube models, digital tools, and grid paper	visualise and draw nets for prisms with a fixed cross section	› physical models or sketches, created using measurement tools › nets.
transform 2D shapes, including composite shapes, by resizing by a whole number or unit fraction of less than one	recognise the invariant properties of 2D and 3D shapes under different transformations	› resizing using a centre of enlargement › 2D shapes and squared paper › tracing paper to predict and test transformations
use the mathematical processes to: – investigate the meaning of kowhaiwhai patterns and other symbols from te ao Māori, and describe the use of transformations in these patterns – explain which properties of a shape will be affected by a given transformation		

Pathways

interpret and communicate the location of positions and pathways using coordinates, angle measures, and the 8 main and halfway compass points (e.g., 45° E from N is NE)	use map scales, compass points, distance, and turn to interpret and communicate positions and pathways in coordinate systems and grid reference systems	› demonstrations using 4-digit grid references and scale to calculate actual distance › maps of familiar and unfamiliar locations › protractors and angle notation › diagrams of compass points › coordinate and grid references.
use the mathematical processes to: – connect map scales to proportional reasoning – connect angles and using a protractor with compass points – investigate the most efficient route between two destinations.		

Phase Three Statistics		
Problem		
Year 7	**Year 8**	**Teaching Methods/Representations**
investigate, using multivariate datasets, summary, comparison, time-series, and relationship situations for paired categorical data by: – posing investigative questions about local community matters – making predictions or assertions about expected findings		Demonstrate posing investigative questions, and support students to write their own questions.
use the statistical processes to: – represent summary, comparison, relationship, and time-series investigative questions – investigate a broad area of interest before fine-tuning a specific investigative question		
Plan		
plan how to collect or source data to answer investigative questions, including – determining or identifying the variables needed – planning how to collect data for each variable (e.g., how to measure them when collecting) or finding out how provided data was collected – identifying the group of interest or who the data was collected from – building awareness of ethical practices by strategic questioning of data collection methods		Demonstrate making a plan for how to collect and sort the data needed to answer an investigative question.
Data		
collect data, including – checking for errors, following up and correcting them when possible – creating data dictionaries that include information for others about the context	source ready-to-use data, and provide information about the variables using provided data dictionaries	Demonstrate a range of data collection methods and what errors in data ‘look like’.
use the statistical processes to: – represent data using a range of tools (e.g., spreadsheets, recording sheets) – investigate secondary data – explain errors in data and justify why they are errors		
Analysis		
create and describe data visualisations for summary, comparison, relationships (paired categorical), and time-series investigations, including features and context in descriptions of distributions	create and describe data visualisations for summary, comparison, relationships, and time-series investigations, using multiple visualisations to provide different views of the data and including features and context in descriptions of distributions	Demonstrate constructing and analysing data visualisations.
use the statistical processes to: – represent data using dot plots, bar graphs, frequency tables, time-series graphs, two-way tables or graphs, scatter plots, fractions, proportions, and percentages – investigate how different data visualisations show different features of data and give different information – explain and justify patterns, trends, and features of data visualisations		

Phase Three Statistics		
Conclusion		
Year 7	**Year 8**	**Teaching Methods/Representations**
communicate findings in context to answer an investigative question, using evidence from analysis and comparing findings to initial predictions or assertions and existing knowledge of the world	communicate findings in context to answer an investigative question, using evidence from analysis, considering possible explanations for findings, and comparing findings to initial predictions or assertions and existing knowledge of the world	Demonstrate making statements about data and matching them with investigative questions. Demonstrate reflecting on findings.
use the statistical processes to: – connect statements with data visualisations to answer an investigative question – investigate appropriate situations – explain findings, and justify initial predictions or assertions given the findings		
Statistical Literacy		
examine the findings of others to check if their claims or statements are supported by the data visualisations they use	examine the data-collection methods, data visualisations, and findings of others' statistical investigations to see if their claims are reasonable	Demonstrate identifying misleading data, matching data visualisations, and checking the claims of investigations.
use the statistical processes to explain and justify critiques of data visualisations and collection methods.		

Phase Three Probability

Probability Investigations

Year 7	Year 8	Teaching Methods/Representation
plan and conduct probability experiments for chance-based situations, including undertaking a large number of trials using technology, by: – posing investigative questions – identifying outcomes for the investigative question posed and anticipating what might happen – deciding on the number of trials, the tools to be used, and the recording method – collecting and recording data – creating data visualisations for the distribution of observed outcomes and (year 8) for all possible outcomes for theoretical probability models where they exist – describing what these visualisations show – finding the probability estimates for the different outcomes – proposing possible theoretical outcomes and associated probabilities for situations where no theoretical model exists – identifying similarities and differences between their findings and those of others – reflecting on anticipated outcomes – identifying similarities and differences between findings from probability experiments and associated theoretical probabilities, as appropriate		Use probability experiments such as taking coloured blocks out of a bag, drawing a card, and flipping a coin, comparing theoretical with experimental probability. Demonstrate creating data visualisations that show outcomes.

Critical Thinking in Probability

agree or disagree with others' conclusions by interrogating their probability experiments		Demonstrate and support students to interpret data and construct a response, using sentence starters or writing frames. Use 'true or false' or 'odd one out' challenges, asking students to support their ideas with reasons.
	agree with or challenge claims and identify misconceptions in relation to chance-based situations	

use the statistical processes to:
– represent outcomes using systematic approaches and technology
– connect probabilities with proportional reasoning, fractions, and percentages
– investigate games of chance, patterns in possible outcomes, and theoretical and experimental distributions
– explain and justify probability estimates and claims about chance-based situations.